TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
METHODS

Test 2, 2023
Section One: Calculator Free
Applications of Anti-derivative, FTOC & DRVs

STUDENT'S NAME: Solutions [LAWRENCE]

DATE: Monday 8th May **TIME:** 25 minutes **MARKS:** 28
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 1 **(6 marks)**

Determine

(a) $\int (8x-6)e^{2x^2-3x+1}dx$ $= 2e^{2x^2-3x+1} + c$ (2 marks)

✓ correct integration of e
✓ +c

(b) $\int\left(\sin\frac{3x}{2} - 2x\cos\frac{2\pi}{5}\right)$ $= -\frac{2}{3}\cos\frac{3x}{2} - x^2\cos\frac{2\pi}{5} + c$ (2 marks)

✓ integrating sin term
✓ integrating 2x term

(c) $\frac{d}{dx}\int_x^1 \frac{2}{3t^3-1}dt$ $= -\frac{d}{dx}\int_1^x \frac{2}{3t^3-1} = -\frac{2}{3x^3-1}$ (2 marks)

✓ recognising FTOC
✓ -ve for changing boundaries

Question 2 **(6 marks)**

Determine, with reasoning, whether each of the following represent a discrete random variable.

(a)

x	0	1.5	2	3	4
$P(X=x)$	0.3	0.1	0.4	0.05	0.15

(2 marks)

Yes all $p(x) \geq 0$ ✓

$\Sigma p(x) = 1$ ✓

(b)

x	-2	-1	1	3	5
$P(X \leq x)$	0	0	0.2	0.6	1

(2 marks)

Yes all $p(x) \geq 0$ ✓

$\Sigma p(x) = 1$ ✓ (from CDF)

(c) $P(X = x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, 4....$ (2 marks)

Yes all $p(x) \geq 0$ ✓

$S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 \quad \therefore \Sigma p(x) = 1$ ✓

Question 3 **(5 marks)**

(a) Determine $\frac{d}{dx}(2x\sin(3x))$ (2 marks)

$u = 2x \quad v = \sin 3x$

$u' = 2 \quad v' = 3\cos 3x$

$= 2x(3\cos 3x) + 2(\sin 3x)$

$= 6x\cos 3x + 2\sin 3x$

✓ use of product rule

✓ correct integration of both $2x$ & $\sin 3x$

(b) Using your answer from (a) or otherwise, determine $\int 6x\cos(3x)dx$ (3 marks)

$\int 6x\cos 3x + 2\sin 3x = 2x\sin 3x$

$\int 6x\cos 3x + \int 2\sin 3x = 2x\sin 3x$

$\int 6x\cos 3x \, dx = 2x\sin 3x - \int 2\sin 3x$

$= 2x\sin 3x + \frac{2}{3}\cos 3x + c$

✓ integral statement

✓ split integrals

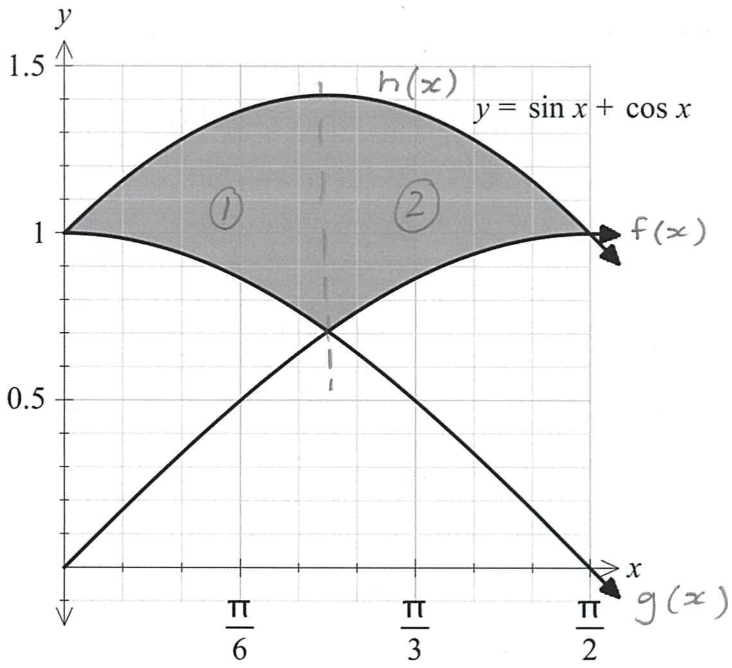
✓ integrates $2\sin 3x$

Question 4 **(4 marks)**

The graph on the right shows three functions:

$f(x) = \sin x$
$g(x) = \cos x$
$h(x) = \sin x + \cos x$

Determine the exact value of the shaded area.



$\sin x = \cos x$

$x = \frac{\pi}{4}$

1 2

$$\text{Area} = \int_0^{\pi/4} \sin x + \cos x - \cos x \; + \int_{\pi/4}^{\pi/2} \sin x + \cos x - \sin x$$

$$= \int_0^{\pi/4} \sin x \; + \int_{\pi/4}^{\pi/2} \cos x$$

$$= \left[-\cos x\right]_0^{\pi/4} + \left[\sin x\right]_{\pi/4}^{\pi/2}$$

$$= \left(-\frac{\sqrt{2}}{2} + 1\right) + \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$= 2 - \sqrt{2} \text{ units}^2$$

✓ determines intersection point

✓ correct addition of integrals with correct boundaries

✓ integrates both functions correctly

✓ substitution for final area

Question 5 **(7 marks)**

(a) Each of the following represent discrete probability functions. Determine the value of k for each.

(i) $P(x) = \frac{1}{k}$; $x = 1, 2, 3, \ldots 12$ (Uniform Distribution) (1 mark)

$k = 12$ ✓

(ii)

x	1	2	3	5	7
$P(X=x)$	$2k$	k	k	$5k$	$6k$

(2 marks)

$\Sigma p(x) = 1$

$2k + k + k + 5k + 6k = 1$

$15k = 1$

$k = 1/15$

✓ shows sum of terms = 1

✓ k value

(b) The random variable X has probability distribution function $p(x)$ defined by $p(x) = \frac{x+2}{k}$ for $x = -1, 0, 1$ and 2.

(i) Determine the value of k. (2 marks)

$\frac{-1+2}{k} + \frac{0+2}{k} + \frac{1+2}{k} + \frac{2+2}{k} = 1$

$\frac{1}{k} + \frac{2}{k} + \frac{3}{k} + \frac{4}{k} = 1$

✓ substituting x values

✓ solving for k

$\therefore k = 10$

(ii) Calculate $P(X = 0 | X \neq 1)$. (2 marks)

$= \frac{P(X=0)}{P(X=-1) + P(X=0) + P(X=2)}$

$= \frac{2/10}{1/10 + 2/10 + 4/10} = 2/7$

✓ correct probability statement

✓ correct probability (simplified)

END OF QUESTIONS

TRINITY COLLEGE

YEAR 12
MATHEMATICS
METHODS

Test 2, 2023
Section Two: Calculator Allowed
Applications of Anti-derivative, FTOC & DRVs

STUDENT'S NAME: ______________________________

DATE: Monday 8th March **TIME:** 25 minutes **MARKS:** 31
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 6 **(4 marks)**

Harry fires an arrow at a target n times. The probability, p, of Harry hitting the target is constant and all shots are independent.

Let X be the number of times Harry hits the target in the n attempts.

The mean of X is 32 and the standard deviation is 4.

(a) State the distribution of X. (1 mark)

Binomial $(X \sim B(n, p))$

✓ Binomial OR $X \sim B(n,p)$

(b) Determine n and p. (3 marks)

$np = 32$

$\sqrt{np(1-p)} = 4$

$\sqrt{32(1-p)} = 4$

$32(1-p) = 16$

$1-p = 1/2$

$p = 1/2$

$\frac{1}{2}n = 32$

$n = 64$

✓ $np =$

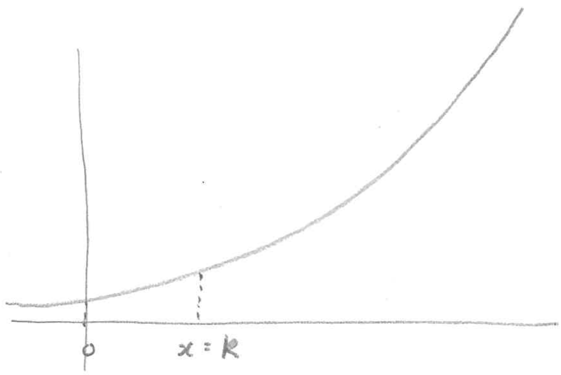
✓ $\sqrt{np(1-p)} =$

✓ solves for n & p.

Question 7 **(4 marks)**

The area bound by the curve $y = e^{x+1} + 2e$ and the x - axis between the values $x = 0$ and $x = k$ is equal to $e^3 - e^2$ square units.

Determine the value of k given $k > 1$.



$$\int_0^k e^{x+1} + 2e = e^3 - e^2$$

✓ correct integral statement & boundaries

$$= \left[e^{x+1} + 2ex\right]_0^k$$

✓ integrates

✓ subs in k & 0

✓ solves for k

$$= (e^{k+1} + 2ek) - (e + 0) = e^3 - e^2$$

use CAS to solve

$k = 1.19$

Question 8 **(7 marks)**

At time $t = 0$, a small body P is at the origin O and is moving with a velocity of 18 ms^{-1}. The acceleration of P for $t \geq 0$ is given by $a = \frac{-3}{\sqrt{t+4}}\, ms^{-2}$.

(a) Determine the velocity of P when $t = 5$. (4 marks)

$$v = \int a \, dt$$

$$= -6\sqrt{t+4} + c$$

when $t=0$ $v=18$

$$18 = -6\sqrt{4} + c$$

$$c = 18 + 12 = 30$$

$$v = -6\sqrt{t+4} + 30$$

$$v(5) = 30 - 18 = 12\ ms^{-2}$$

✓ integrates for $v(t) + c$

✓ finds 'c'

✓ correct $v(t)$

✓ subs in $v(5)$

(b) Determine the distance of P from O at the instant P is stationary. (3 marks)

$$v = 0 = 30 - 6\sqrt{t+4}$$

$$30 = 6\sqrt{t+4}$$

$$5 = \sqrt{t+4}$$

$$t = 21$$

$$OP = \int_0^{21} 30 - 6\sqrt{t+4}\ dt$$

use CAS

$$= 162\ m$$

✓ solves $v=0$ for t

✓ correct integral & boundaries

✓ finds distance

Question 8 **(9 marks)**

Studies in Britain have recorded that 1 in 100 eight-year-old children need at least one tooth removed caused by sugary drinks and severe tooth decay.
A typical primary school class of 24 eight-year-olds are investigated for the need to remove at least one tooth.

$X \sim B(24, 0.01)$

Determine the probability of:

(a) 2 students needing at least one tooth removed. (1 mark)

$P(X=2) = 0.0221$

✓ correct probability

(b) No students requiring the removal of any teeth. (1 mark)

$P(X=0) = 0.7857$

✓ correct probability

(c) At least one student requiring the removal of at least one tooth. (2 marks)

$P(X \geq 1) = 0.2143$

✓ correct statement

✓ correct probability

(d) Less than 4 students requiring the removal of at least one tooth given that at least one student required tooth removal. (2 marks)

$$\frac{P(1 \leq X < 4)}{P(X \geq 1)} = \frac{0.2142}{0.2143} = 0.9997$$

✓ correct statement

✓ correct probability

Of the thirteen-year-olds in Britain requiring tooth removal, the probability of them requiring just one tooth out of their 32 permanent teeth removed is 5%.

$n = 32$ $P(X=1) = 0.05$

(e) Calculate the probability of a permanent tooth in a thirteen-year-old needing removal. (3 marks)

$p = ?$

$${}^{32}C_1 \, p^1 (1-p)^{31} = 0.05$$

$$32p(1-p)^{31} = 0.05$$

use CAS

$$p = 0.1337$$

✓ recognises $P(X=1)=0.05$

✓ correct statement

✓ solves for p

Question 9 **(7 marks)**

The discrete random variable X has the following probability distribution:

x	1	2	3	4	5
$P(X = x)$	0.1	a	0.3	0.25	b

(a) Determine the values of a and b if the expected value, $E(X) = 3.3$ (3 marks)

$\Sigma p(x) = 1$

$a + b = 0.35$

$E(X) = 3.3$

$0.1 + 2a + 0.9 + 1 + 5b = 3.3$

use CAS

$a = 0.15$

$b = 0.2$

✓ Σpx statement
✓ $E(X)$ statement
✓ solves for a & b

(b) Determine the variance, $Var(x)$. (2 marks)

use CAS $\sigma x = 1.23$

$\therefore Var(x) = 1.51$

✓ σx
✓ $Var(x)$

(c) State the value of $E(X + 5)$ $= 8.3$ ✓ (1 mark)

(d) State the value of $Var(5 - 2X)$ $= 4(1.51)$ (1 mark)

$= 6.04$ ✓

END OF QUESTIONS